# Probability for the outcome of a chess game based on rating

by
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## Introduction

If you want to explore how different tournament systems, or how tie-break systems behave with different combination of players, it may be useful to simulate tournaments with different types of parameters. In such simulated tournament you may ask what the probability for the outcome of a single chess game given the rating performance of the two players. The goal of this paper is to establish a probability function that may be applied on all games in a simulated tournament.

## Ground truth

The FIDE server contains results from all rated tournament from the last years. I have search through all tournament played from August 2010 to October 2014 (Tournament 50000-100000). A few children tournaments with very strange result were excluded (Tournament: 84129, 86349-86364, 88121-88128, 97263, 97272).

The file allgames.txt compressed to http://www.milvang.no/spp/allgames.zip contains lines like this:

# Tournament: 53305, Player: 316415, Rating: 2008
b;1948;2008;0.5
w;2008;1961;1.0
b;2130;2008;0.0
w;2008;2176;0.5
b;2267;2008;1.0
w;2008;2172;1.0
w;2008;2291;0.5
b;2245;2008;1.0
w;2008;2150;0.0

The line starting with # is a comment such that is possible to find the source of the data. This is lines are from tournament 53305, player Id 316415 with FIDE rating 2008

For each game he played in the tournament, there is a line with
color;white rating;black rating;result
result may be: 0.0, 0.5, 1.0, +, -

Then I have analyses all games for white (all game results are for both players, so no need to read it twice), and counted result for white win, draw, black win, and plotted the distribution for different outcome as a function of rating difference $R_d$.

$R_d$ = Rating difference = White rating – Black rating

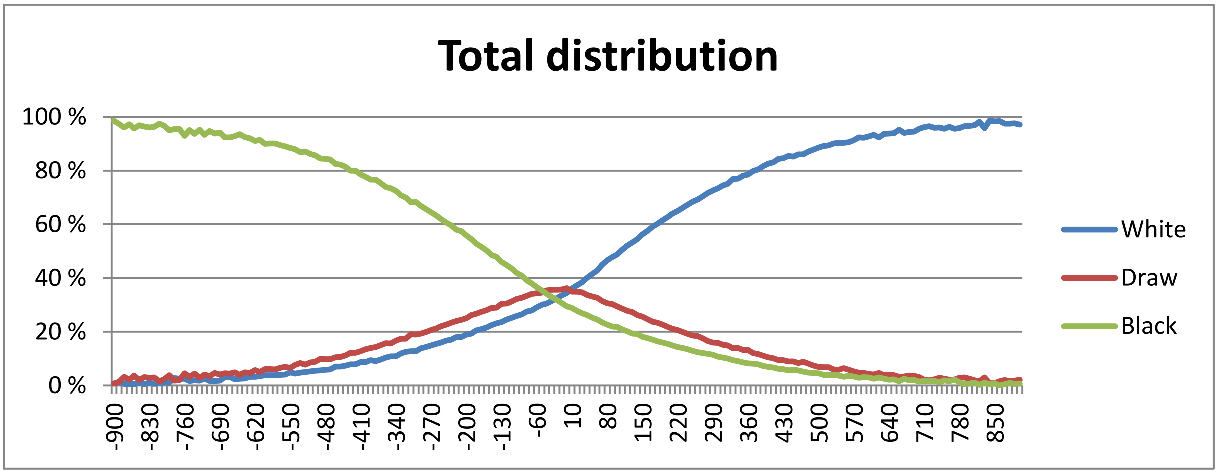
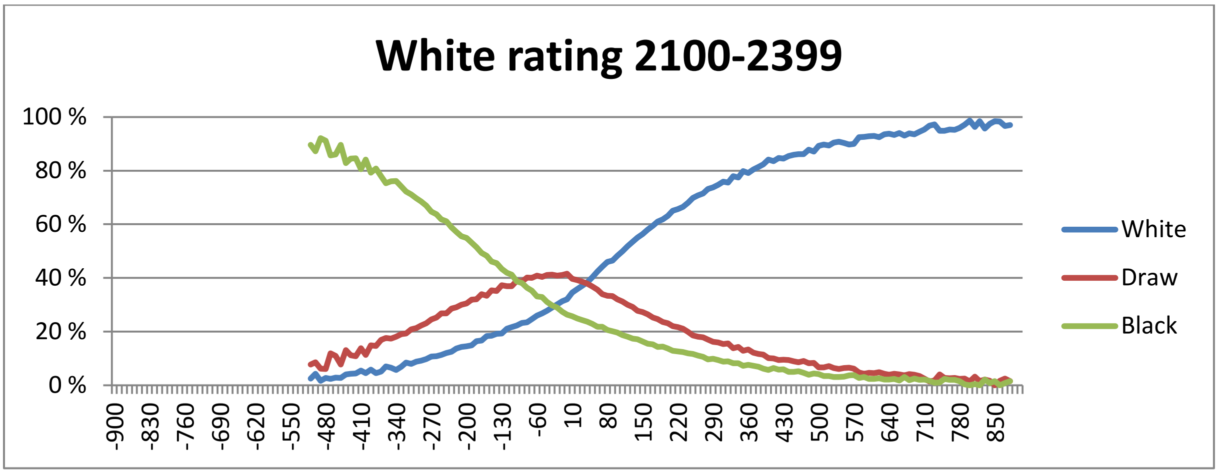
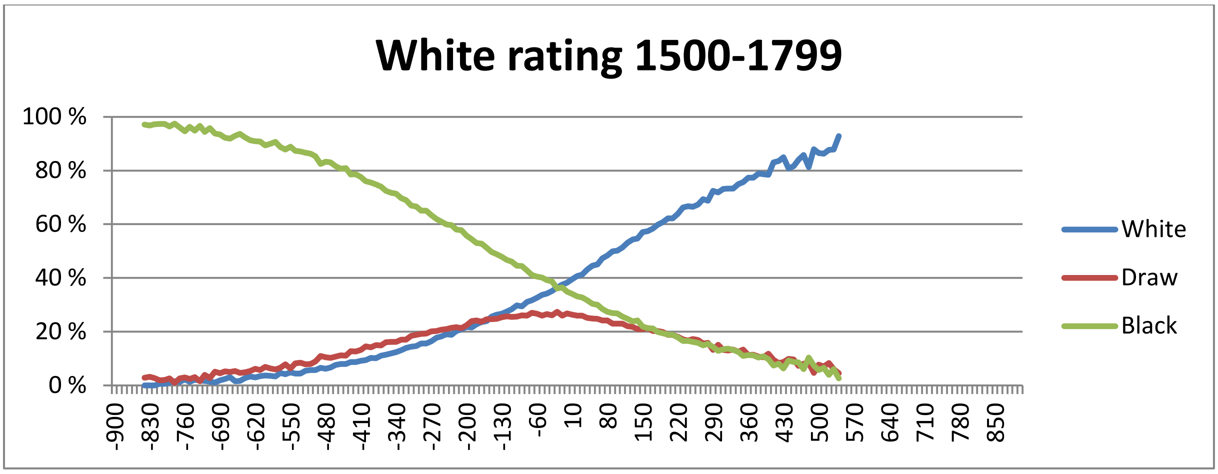



Fig 1 shows the distribution of 3912831 games. We can read that if we count up all games where players with equal strength has met (+-5 rating points) white won in 34.42% of the games, black won in 29.39% of the games and the rest **36.19%** ended with draw.




In games where Whites rating is in the range 2100-2399 and players with equal strength has met (+-5 rating points) **41.56%** of the games ended with draw.

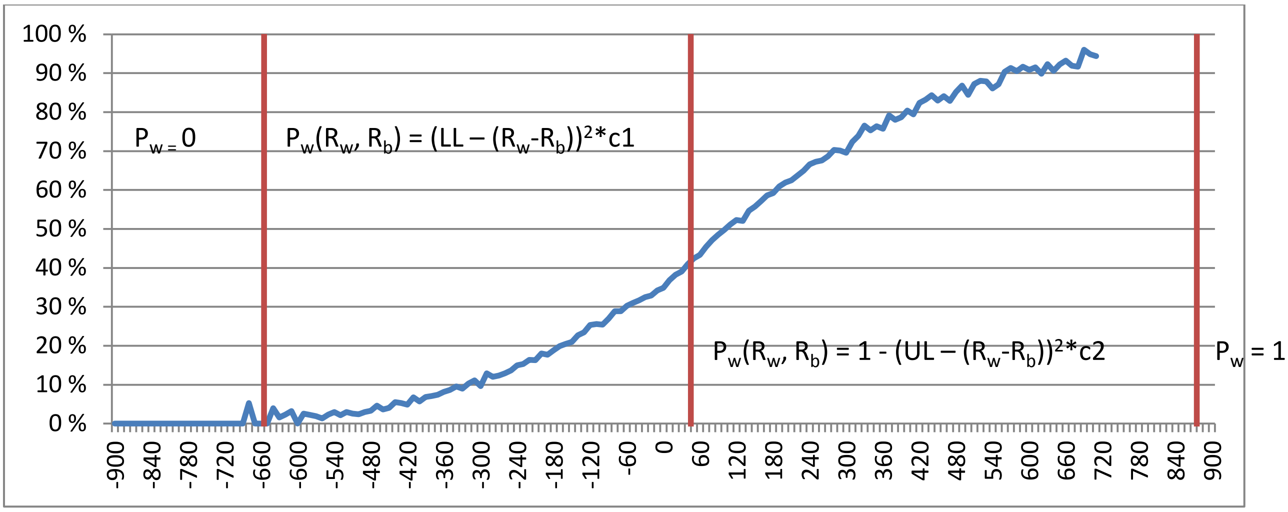



In the range 1500-1799 and players with equal strength has met (+-5 rating points) only **26.86%** of the games ended with draw.

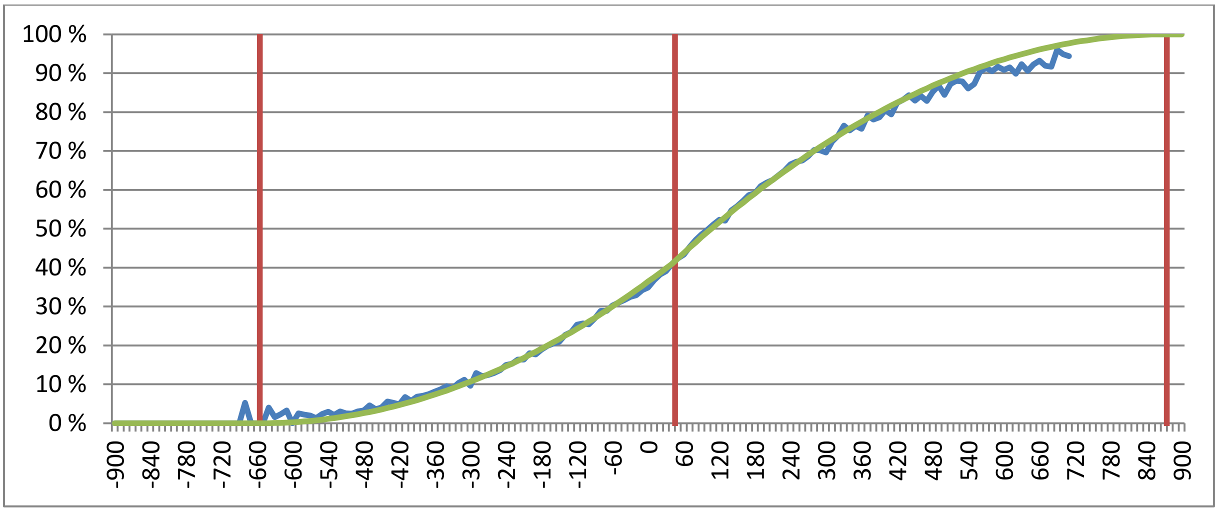
## Function approximation

There are different choices of functions used for approximation of data sets. A common choice is polynomial functions. For given value of Wr (white rating) we define three constants LL (Lower limit), CL (center limit) and UL (upper limit). The probability for white win, Pw, can be expressed with four functions:

| | |
|---|---|
| $P_w(R_w, R_b) = 0$ | for $R_w\text{-}R_b < LL$ |
| $P_w(R_w, R_b) = (LL - (R_w\text{-}R_b))^2$ | for $LL <= R_w\text{-}R_b <= CL$ |
| $P_w(R_w, R_b) = 1 - (UL - (R_w\text{-}R_b))^2$ | for $CL <= R_w\text{-}R_b <= UL$ |
| $P_w(R_w, R_b) = 1$ | for $UL < R_w\text{-}R_b$ |




With Wr=2000, LL=-655, CL=40, UL = 876, c1 = $84.869*10^{-8}$, c2 = $84.427*10^{-8}$, we get:



We can do the same for probability for black Pb, and for probability for draw Pd,
we have Pd = 1 – Pw – Pb

# Parameter approximation based on the data set (Solution)

## Probability for white win Pw based on White/Black rating Wr and Br

WCL = 40

Rm = (Wr + Br) / 2

Rd = Wr - Br

WCV = 0.45 - 0.1 * $(Rm - 1200)^2 / 1200^2$ if Rm > 1200, 0.45 if Rm <= 1200

WLL = -1492 + Rm * 0.391

WUL = 1691 - Rm * 0.428

**If Rd < WLL**

Pw = 0

**If WLL <= Rd <= WCL**

Pw = WCV * $(Rd - WLL)^2 / (WCL - WLL)^2$

**If WCL <= Rd <= WUL**

Pw = 1 – (1-WCV) * $(Rd - WUL)^2 / (WCL - WUL)^2$

**If Rd > WUL**

Pw = 1

## Probability for black win Pb based on White/Black rating Wr and Br

BCL = -80

Rm = (Wr + Br) / 2

Rd = Wr - Br

BCV = 0.46 - 0.13 * $(Rm - 1200)^2 / 1200^2$ if Rm > 1200, 0.46 if Rm <= 1200

BLL = -1753 + Rm * 0.416

BUL = 1428 - Rm * 0.388

**If Rd < BLL**

Pb = 1

**If BLL <= Rd <= BCL**

Pb= 1 – (1-BCV) * $(Rd - BLL)^2 / (BCL - BLL)^2$

**If BCL <= Rd <= BUL**

Pb = BCV * $(Rd - BUL)^2 / (BCL - BUL)^2$

**If Rd > BUL**

Pb = 0

## Probability for draw Pd based on White/Black rating Wr and Br

Pd = 1 - Pw - Pb

## Example

White rating = 2467
Black rating = 2344

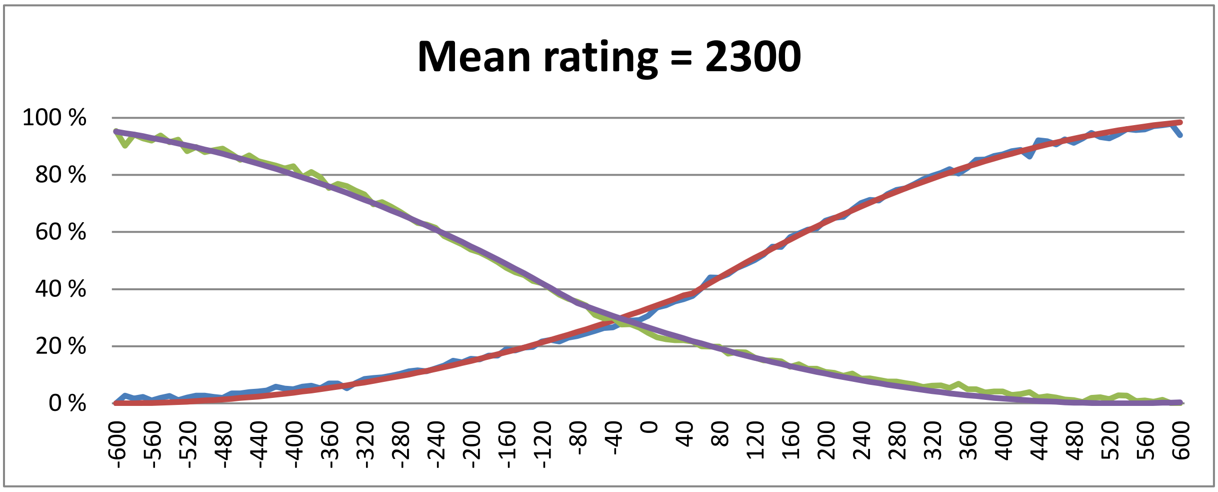
| | |
|---|---|
| WCL | 40 |
| Rm | 2405.5 |
| Rd | 123 |
| WCV | 0.349 |
| WLL | -551.450 |
| WUL | 661.446 |
| **Pw** | **0.511** |
| | |
| BCL | -80 |
| Rm | 2405.5 |
| Rd | 123 |
| BCV | 0.329 |
| BLL | -752.312 |
| BUL | 494.666 |
| **Pb** | **0.138** |
| | |
| **Pd** | **0.351** |

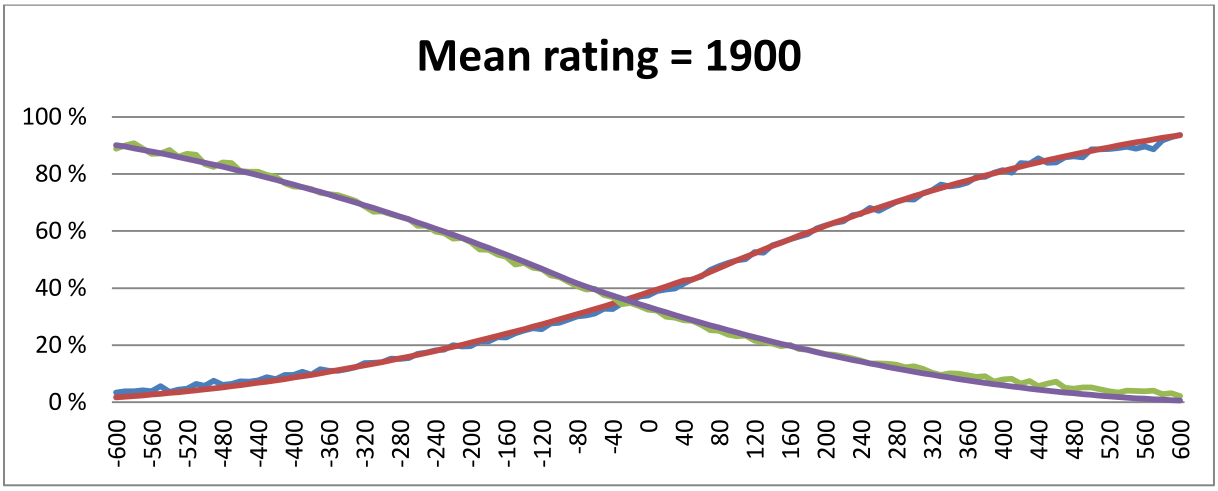
## Verification of the solution

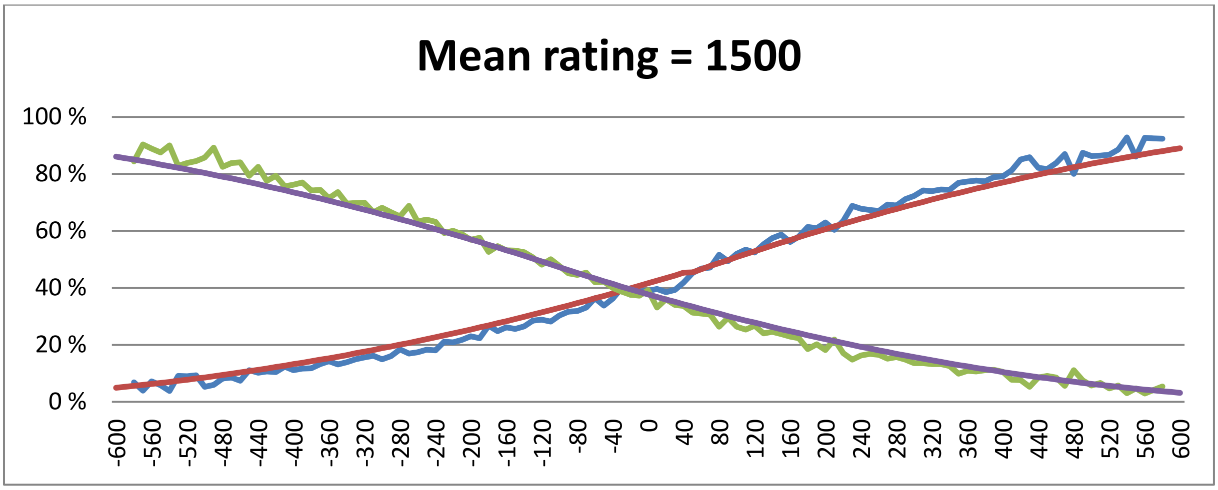
I have extracted all games where the mean rating (white rating + black rating)/2 equal 1500, 1900 and 2300, and plotted the probability for white win and white win as a function of rating difference (white rating – black rating).

Blue – Percentage white win in data set
Red – Probability for white win according to the formulas
Green - Percentage black win in data set
Violet – Probability for black win according to the formulas

Since probability for draw equal 1 – Pw – Pb both for the dataset and for the formulas these values are not plotted.


Mean rating = 2300
100 %
80 %
60 %
40 %
20 %
0 %
-600 -560 -520 -480 -440 -400 -360 -320 -280 -240 -200 -160 -120 -80 -40 0 40 80 120 160 200 240 280 320 360 400 440 480 520 560 600


Mean rating = 1900
100 %
80 %
60 %
40 %
20 %
0 %
-600 -560 -520 -480 -440 -400 -360 -320 -280 -240 -200 -160 -120 -80 -40 0 40 80 120 160 200 240 280 320 360 400 440 480 520 560 600


Mean rating = 1500
100 %
80 %
60 %
40 %
20 %
0 %
-600 -560 -520 -480 -440 -400 -360 -320 -280 -240 -200 -160 -120 -80 -40 0 40 80 120 160 200 240 280 320 360 400 440 480 520 560 600

## Conclusion and discussion

There have been a lot of decisions to take in this work. It's always a tradeoff between the complexity in the model, and the accuracy in the calculation. A very important choice was the value for VCL and BCL. VCL and BCL is an approximation of where the second derivative is zero. My choice was to simplify the model by using a fixed value for white (40) and a fixed value for black (-80). It seems that the calculation is quite robust with these values. When I did the choice to approximate the function with quadratic functions, the job was simply to find parameters that minimized the mean square error.

For all parameters I have tested the model seems to work very well.

Oslo 12th January 2015, Otto Milvang

# Appendix A, Implementation in C#

```
static private Random ran;

// DrawResult - Draw the result of a game based on rating
// Returns: 'w' , 'b' or '='
// Input - whiteRating, blackRating
public static char DrawResult(int whiteRating, int blackRating)
{
    double pw = 1.0, pd = 1.0, pb = 0.0; // probability white, draw, black
    if (ran == null) ran = new Random(47);

    double rm = (whiteRating + blackRating) / 2.0;
    double rd = (whiteRating - blackRating);
    double rmq = (rm > 1200.0) ? ( (rm-1200.0)/1200.0 )  : 0.0;

    double wcl =  40.0;
    double wcv = 0.45 - 0.1 * rmq * rmq;
    double wll = -1492.0 + rm *0.391;
    double wul = 1691.0 - rm *0.428;

    double bcl = -80.0;
    double bcv = 0.46 - 0.13 * rmq * rmq;
    double bll = -1753 + rm * 0.416;
    double bul = 1428 - rm * 0.388;

    double wf1 = (rd - wll) / (wcl - wll);
    double wf2 = (rd - wul) / (wcl - wul);
    double bf1 = (rd - bll) / (bcl - bll);
    double bf2 = (rd - bul) / (bcl - bul);

    if (rd < wll)                        pw = 0.0;
    else if (wll <= rd && rd <= wcl) pw = wcv * wf1 * wf1;
    else if (wcl <= rd && rd <= wul) pw = 1.0 - (1.0 - wcv) * wf2 * wf2;
    else if (rd > wul)                   pw = 1.0;

    if (rd < bll)                        pb = 1.0;
    else if (bll <= rd && rd <= bcl) pb = 1.0 - (1.0 - bcv) * bf1 * bf1;
    else if (bll <= rd && rd <= bul) pb = bcv * bf2 * bf2;
    else if (rd > bul)                   pb = 0.0;

    pd = 1.0 - pw - pb;

    double r = ran.NextDouble();
    if (r < pw) return 'w';
    if (r < pw + pb) return 'b';
    return '=';
}
```